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I Semester M.Sc. Degree Examination, February 2019
(CBCS Scheme)
MATHEMATICS
Paper – M 101 T : Algebra – I

Time : 3 Hours Max. Marks : 70

Instructions : i) Answer any five questions.
ii) All questions carry equal marks.

1. a) Show that $S_n/A_n = \{1, -1\}$ where S_n is a symmetric group and A_n is alternating group.
   b) If N and M are normal subgroup of a group G, prove that $NM/M \approx N/N \cap M$.
   c) State and prove the Cayley's theorem for permutation groups. (4+5+5)

2. a) Let G be a finite group and S is finite G–set. If $x \in S$ then show that $O(G_x) = O(G)/O(stab(x))$.
   b) Derive the class equation for finite groups.
   c) By using generator-relation form of S_3, verify the class equation of S_3, where S_3 is a symmetric group. (5+4+5)

3. a) State and prove the Cauchy's theorem for abelian group.
   b) Show that the number of p-sylow subgroups of G for a given prime is congruent to 1 modulo p.
   c) Show that a group of order pq with p and q are distinct primes such that $p < q$ and $q \not\equiv$ (mod p) is abelian. (5+5+4)

4. a) Define a solvable group. Prove that every subgroup of a solvable group is solvable.
   b) State and prove Jordan-Holder theorem. (7+7)

5. a) Let R be a commutative ring with unity whose only ideals are {0} and R itself. Then prove that R is field.
   b) Show that the homomorphism ϕ of R onto R^1 is an isomorphic if and only if $Ker\phi = \{0\}$.
   c) Let R and R^1 be rings and ϕ is a homomorphism of R and R^1 with kernel U. Then show that $R^1 \approx R/U$. (5+5+

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6. a) Define a principal ideal ring. Show that a ring Z of integer is a principal ideal ring.

b) Define maximal ideal of a ring. If R is commutative ring with element and M is an ideal of R, then show that M is a maximal ideal of R if and only if R/M is a field.

c) Show that any two isomorphic integral domain have isomorphic quotient fields. (5+5+4)

7. a) Let R be an Euclidean ring. Show that any ideal $A = (a_0)$ is maximal ideal in R if and only if a_0 is a prime element of R.

b) Show that every Euclidean ring is a principal ideal ring.

c) State and prove the unique factorization theorem. (5+4+5)

8. a) Prove that deg (fg) = deg (f) + deg (g) for f, g $\in$ R [x]. Further if R is an integral domain, then show that R [x] is also an integral domain.

b) State and prove the Gauss lemma.

c) If p is a prime number, prove that the polynomial $x^n - p$ is irreducible over Q. (5+5+4)